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JET-BOUNDARY CORRECTIONS TO A YAWED MODEL IN A CLOSED RECTANGULAR WIND TUNNEL

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SUMMARY

General formulas are derived for determining the jet-boundary corrections to the angle of attack, the drag coefficient, the downwash, the pitching-, rolling-, and yawing-moment coefficients of complete powered yawed models, and to the increments of rolling- and yawing-moment coefficients due to the ailerons on yawed models. Numerical values of the jet-boundary induced velocities and the corresponding corrections are calculated for a typical powered model yawed 20° in a 7- by 10-foot closed wind tunnel. The results of the calculations are compared with the results for the unyawed model.

The calculations indicate that, for this particular model-tunnel combination and for a constant immersion of the tail in the slipstream, the corrections to the angle of attack, drag coefficient, pitching-moment coefficient, and downwash are all about 6 percent greater at 20° yaw than at the same lift coefficient at zero yaw. Because the immersion of the tail in the slipstream is changed by yaw, the downwash and pitching-moment-coefficient corrections are about 20 to 25 percent greater at 20° yaw than at zero yaw for rated power operation of the model at unit lift coefficient.

The correction to the rolling-moment coefficient for the model at 20° yaw is approximately 15 percent lower than the correction for the unyawed model. The correction to the yawing-moment coefficient for the aileron on the leading wing is 8 percent lower for the yawed model than for the unyawed model, but the correction for the aileron on the trailing wing is about 20 percent greater for the yawed model than for the unyawed model.

The correction at unit coefficient and at 20° yaw to the yawing-moment coefficient of the complete model with or without power is about 5 percent of the yawing-moment coefficient of the model.

INTRODUCTION

The influence of the jet boundaries upon the downwash at the wing and behind the wing of unyawed models has been extensively investigated (references 1 to 3). The assumption is usually made that the same jet-boundary corrections may be applied to a yawed model as are applied to an unyawed model. In order to investigate the validity of this assumption, the methods of determining the jet-boundary corrections were extended to cover the case of yawed models. General formulas were developed for the jet-boundary corrections to the angle of attack, drag coefficient, downwash angle, pitching-, rolling-, and yawing-moment coefficients, and to the increments of rolling-and yawing-moment coefficients caused by aileron deflection. By means of these formulas the jet-boundary corrections were calculated for a typical powered model yawed 20° in a 7- by 10-foot closed rectangular wind tunnel and the results were compared with the results for the unyawed model.

SYMBOLS

Γ	vortex strength
C_L	wing lift coefficient, wind axes
c_l	section lift coefficient
C_Y'	lateral-force coefficient, wind axes
C_l'	rolling-moment coefficient, wind axes
ψ	angle of yaw, degrees
V	velocity parallel to X' axis, wind axes
v	induced velocity parallel to Y' axis, wind axes
w	induced velocity parallel to Z' axis, wind axes
q	dynamic pressure
ρ	mass density of air
σ	angle of sidewash, radians

a	tunnel width
h	tunnel height
C	tunnel area (ah)
m	integer defining number of images in Z' direction
n	integer defining number of images in Y' direction
S_w	wing area
S_f	area of vertical tail
c	chord
b	wing span
A	aspect ratio (b^2/S)
λ	taper ratio
l	distance from lifting line to three-quarter-chord point of tail, body axes
$\lvert a_1 \rvert$	absolute distance from plane of symmetry to inboard aileron tip or dihedral juncture, body axes
$\lvert a_2 \rvert$	absolute distance from plane of symmetry to outboard aileron tip or dihedral juncture, body axes
y	distance from plane of symmetry, body axes
y_1	distance from plane of symmetry to trailing vortex location ($\lvert y_1 \rvert$ = bound vortex semispan), body axes
δ	jet-boundary correction factor
$\Delta\alpha$	correction to angle of attack
$\Delta\epsilon$	correction to downwash angle
ΔD_i	correction to the induced drag, wind axes
ΔC_{D_i}	correction to the induced drag coefficient, wind axes
$\Delta C_l'$	correction to the rolling-moment coefficient, wind axes

$\Delta C_{n_i}'$ correction to the induced yawing-moment coefficient, wind axes

ΔC_m correction to the pitching-moment coefficient

$\frac{\partial c_l}{\partial \alpha}$ slope of the section lift curve, per degree

$\frac{\partial C_N}{\partial \alpha}$ slope of the vertical-tail normal-force curve, per degree

$\frac{\partial C_m}{\partial i_t}$ stabilizer effectiveness, change in pitching-moment coefficient per degree change in stabilizer angle

$j = (-1)^n \left| y_1 \right| \cos \psi$

$p = -na + y \cos \psi - l \sin \psi$

$g = y \sin \psi + l \cos \psi$

$r = p \cos \psi + g \sin (-1)^n \psi$

$k = g \cos \psi - p \sin (-1)^n \psi$

$z = mh$

$\frac{r}{r_s}, \theta$ position of point with respect to center line of a two-dimensional slipstream in terms of the slipstream radius, r_s

Subscripts:

b basic load

l dihedral or aileron load on leading wing

t dihedral or aileron load on trailing wing

s slipstream

o free stream

w wing

f vertical tail

CALCULATION METHODS

Jet-Boundary-Induced Upwash Velocity

General method.- In order to simplify the calculations of the jet-boundary-induced upwash velocity, the yawed wing is replaced by a series of skewed horseshoe-type vortices. The loading of the yawed wing consists of a basic loading extending across the effective span of the wing and a pair of uniform dihedral loadings over the portions of the wing with dihedral. A yawed wing with positive dihedral normally has the leading wing loaded positively (up load) and the trailing wing loaded negatively (down load). The upwash velocity is computed, however, as if both loads were positive, the sign being taken care of in the calculation of the actual corrections. The loads due to aileron deflection are similar to the dihedral loads and the same general upwash velocity formulas apply to both cases.

The bound vortices of these various loadings are assumed to be yawed the same amount as the wing, and the trailing vortices are assumed to extend uniformly downstream parallel to the free-stream velocity. The interference between the wing and fuselage pressure distributions is neglected in the analysis.

It is known that the jet boundaries impose certain restrictions on the air flow around a model and that the boundary conditions may be satisfied by replacing the jet boundaries by a doubly infinite pattern of images of the model vortex system (reference 1). The image system required to satisfy the boundary conditions for a yawed model in a closed rectangular wind tunnel (zero normal velocity at the walls) is illustrated in figure 1. Figure 1(a) shows the image system for the basic loading; figure 1(b), the loading representing an aileron or dihedral on the leading wing; and figure 1(c), the loading representing an aileron or dihedral on the trailing wing. The lifting line and the tail are assumed to lie in the horizontal plane of symmetry of the tunnel and to yaw with the model.

Basic loading.- The induced upwash velocity caused by the images (fig. 1(a)) of the basic load horseshoe-type vortex of semispan equal to $|y_1|$ is given by the following equation, derived from the Biot-Savart rule for calculating the induced velocity due to vortices:

$$\left(\frac{w}{\Gamma}\right)_b = \frac{1}{4\pi}\sum_n\sum_m (-1)^m \left\{\left[\frac{k}{k^2+z^2}\left(\frac{r-|y_1|}{\sqrt{(r-|y_1|)^2+z^2+k^2}} - \frac{r+|y_1|}{\sqrt{(r+|y_1|)^2+z^2+k^2}}\right)\right]\right.$$

$$+\frac{(-1)^n(p-j)}{(p-j)^2+z^2}\left[1+\frac{g-|y_1|\sin\psi}{\sqrt{(p-j)^2+z^2+(g-|y_1|\sin\psi)^2}}\right]$$

$$\left.-\frac{(-1)^n(p+j)}{(p+j)^2+z^2}\left[1+\frac{g+|y_1|\sin\psi}{\sqrt{(p+j)^2+z^2+(g+|y_1|\sin\psi)^2}}\right]\right\} \qquad (1)$$

where

$j = (-1)^n \; |y_1| \; \cos\psi$

$p = -na + y\cos\psi - l\sin\psi$

$g = y\sin\psi + l\cos\psi$

$r = p\cos\psi + g\sin(-1)^n\psi$

$k = g\cos\psi - p\sin(-1)^n\psi$

$z = mh$

and the summations include all combinations of the integers n and m except the combination $n = 0$, $m = 0$.

Equation (1) for $\psi = 0^\circ$ reduces to the sum of equations (20) and (21) of reference 3 for the upwash velocity at the tail of the unyawed wing.

Dihedral or aileron loading on leading wing.- The induced upwash velocity caused by the images (fig. 1(b)) of the dihedral or aileron load on the leading wing of a yawed model is given by the following equation:

$$\left(\frac{w}{\Gamma}\right)_t = \frac{-1}{4\pi}\sum_n \sum_m (-1)^n(-1)^m \left\{ \frac{k}{k^2+z^2}\left[\frac{r+(-1)^n|a_2|}{\sqrt{(r+(-1)^n|a_2|)^2+z^2+k^2}} - \frac{r+(-1)^n|a_1|}{\sqrt{(r+(-1)^n|a_1|)^2+z^2+k^2}}\right]\right.$$

$$+\frac{p+(-1)^n|a_2|\cos\psi}{(p+(-1)^n|a_2|\cos\psi)^2+z^2}\left[1+\frac{g+|a_2|\sin\psi}{\sqrt{(p+(-1)^n|a_2|\cos\psi)^2+z^2+(g+|a_2|\sin\psi)^2}}\right]$$

$$\left.-\frac{p+(-1)^n|a_1|\cos\psi}{(p+(-1)^n|a_1|\cos\psi)^2+z^2}\left[1+\frac{g+|a_1|\sin\psi}{\sqrt{(p+(-1)^n|a_1|\cos\psi)^2+z^2+(g+|a_1|\sin\psi)^2}}\right]\right\} \quad (2)$$

<u>Dihedral or aileron loading on trailing wing</u>.- The induced-upwash velocity caused by the images (fig. 1(c)) of the dihedral or aileron load on the trailing wing of a yawed model is given in the following equation:

$$\left(\frac{w}{\Gamma}\right)_t = \frac{1}{4\pi}\sum_n \sum_m (-1)^n(-1)^m \left\{ \frac{k}{k^2+z^2}\left[\frac{r-(-1)^n|a_2|}{\sqrt{(r-(-1)^n|a_2|)^2+z^2+k^2}} - \frac{r-(-1)^n|a_1|}{\sqrt{(r-(-1)^n|a_1|)^2+z^2+k^2}}\right]\right.$$

$$+\frac{p-(-1)^n|a_2|\cos\psi}{(p-(-1)^n|a_2|\cos\psi)^2+z^3}\left[1+\frac{g-|a_2|\sin\psi}{\sqrt{(p-(-1)^n|a_2|\cos\psi)^2+z^2+(g-|a_2|\sin\psi)^2}}\right]$$

$$\left.-\frac{p-(-1)^n|a_1|\cos\psi}{(p-(-1)^n|a_1|\cos\psi)^2+z^2}\left[1+\frac{g-|a_1|\sin\psi}{\sqrt{(p-(-1)^n|a_1|\cos\psi)^2+z^2+(g-|a_1|\sin\psi)^2}}\right]\right\} \quad (3)$$

Equation (2) is the negative of equation (3) if all values of a_1 and a_2 are given negative signs instead of using a_1 and a_2 as absolute quantities.

Jet-Boundary-Induced Sidewash Angle

Vortex system.- The jet-boundary-induced sidewash angle due to the wing vortex system is zero for both the yawed model and the unyawed model provided that the tail and wing are in the horizontal plane of symmetry of the tunnel. The fact that there is no resultant induced sidewash angle for the yawed model is easily seen if the images are considered in pairs symmetrical about the horizontal plane of symmetry of the tunnel; that is, the image pairs n, m and n, -m. The sidewash angle due to these image pairs cancel and, because no sidewash can result from the images for which $m = 0$, there can be no resultant induced sidewash. If either the wing or vertical tail is located off the horizontal plane of symmetry of the tunnel, a small sidewash angle will be induced but may usually be neglected.

Slipstream.- The boundaries of a slipstream induce an additional sidewash velocity inside and outside the slipstream. As the effect of the tunnel jet boundaries upon the slipstream may be obtained by replacing the boundaries by a system of image slipstreams, the induced sidewash angle due to the effect of the tunnel jet boundaries upon the slipstream may be determined by calculating the induced sidewash angle due to each image slipstream and by summing the effects of all image slipstreams. The formula for the induced upwash velocity caused by a pitched slipstream, which is given in reference 3 (equation (8)), may also be used for a yawed slipstream. The equation with symbols for sidewash instead of downwash is

$$\frac{v_s}{V} = \frac{q_s/q_o - 1}{q_s/q_o + 1} \frac{v_o}{V} \frac{\sin 2\sigma_s}{2} \sum_n \sum_m (-1)^n \frac{\cos 2\theta}{\left(\frac{r}{r_s}\right)^2} \tag{4}$$

Vertical tail.- A small jet-boundary-induced sidewash velocity is caused by the loads on the fuselage and vertical tail surfaces. As a first approximation, all of the lateral force (wind axis) acting on a yawed model, except that resulting from the operation of a propeller, may be assumed to be caused by the vertical tail. The jet-boundary-induced sidewash resulting from the lateral force may be roughly estimated by extrapolation of the curves of reference 2. This extrapolation should be made for a tunnel with a height to width ratio equal to the reciprocal of the actual height to width ratio and for a vortex span expressed in percent of the tunnel height. With the value of δ for the tail at the center of the tunnel, the sidewash angle v_f/V in radians is found from the lateral-force coefficient and the wing tunnel areas:

$$\frac{v_f}{V} = \delta \frac{S_f}{C} C_Y' \frac{S_w}{S_f} \frac{V_o}{V} = \delta \frac{S_w}{C} \frac{V_o}{V} C_Y' \qquad (5)$$

Corrections

The corrections are so determined that they are to be added to the measured values to give the corrected values of the various aerodynamic quantities.

Angle of attack.- The angle-of-attack correction will be determined by the usual method of averaging the boundary-induced upwash angle at the lifting line across the span; that is, no refinements in method such as weighting the induced upwash angle according to wing chord or calculating the additional angle of attack correction due to streamline curvature are made in determining the correction.

The correction for the unyawed wing is

$$\Delta\alpha = \frac{57.3SC_L}{4y_1 b} \int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b dy \qquad (6)$$

since $\left(\frac{\Gamma}{V}\right)_b = \frac{SC_L}{4y_1}$ for a uniform loading at zero yaw. The angle-of-attack correction for the yawed wing is determined by an integration of the upwash velocity caused by the basic and dihedral loads for the yawed wing:

$$\Delta\alpha = 57.3 \left\{ \frac{SC_L}{4 \cos\psi\, y_1 b} \int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b dy + \frac{SC_l'}{2 \cos^2\psi \left(|a_2|^2 - |a_1|^2\right)} \left[\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_l dy - \int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_t dy \right] \right\} \qquad (7)$$

since

$$\left(\frac{\Gamma}{V}\right)_b = \frac{SC_L}{4y_1 \cos \Psi}$$

$$\left(\frac{\Gamma}{V}\right)_l = \left(\frac{\Gamma}{V}\right)_t = \frac{1}{2} \frac{C_l' \ Sb}{\cos^2\Psi(|a_2|^2 - |a_1|^2)}$$

and C_l' is the total rolling-moment coefficient about the wind axis due to dihedral. It is assumed that the dihedral extends over only the portion of the wing tip between $|a_1|$ and $|a_2|$.

Induced-drag coefficient.- The correction to the induced drag may be determined from the generalized Kutta-Joukowski law as

$$\Delta D_i = \int_{-b/2}^{b/2} \cos \Psi \ \rho w\Gamma \ dy \tag{8}$$

and the correction to the induced-drag coefficient of the wing is

$$\Delta C_{D_i} = \frac{\rho \cos \Psi}{qS} \int_{-b/2}^{b/2} w\Gamma \ dy \tag{9}$$

The accuracy of the computations will be increased if the actual span load distribution is used to determine the value of Γ in equation (9) rather than the assumed uniform load that was used to calculate the boundary-induced upwash velocity. In order to avoid the confusion of having Γ in the same formula to indicate both the uniform load used for the upwash calculations and the actual section load in the Kutta-Joukowski formula, the formulas in the discussion that follows will be written with the load term $cc_l \ V/2$ substituted for the airfoil section Γ. The influence lines of reference 4 may be used to determine the span load distribution for the basic loading and the dihedral loading. The loading parameter used in reference 4 should be converted to $cc_l \ A \cos \Psi/bC_L$ for the basic load to facilitate the computations of the corrections. One relatively simple method of converting the basic loading parameter is to multiply the values of the load parameter of reference 4 by a constant such that the average value across the span of the loading parameter will be equal to unity. That is, the numerical value of the area under the curve of $cc_l\ A \cos\psi / bC_L$ plotted against y must be equal to b. Similarly, the dihedral (or aileron) loading parameter $cc_l\ A \cos^2\psi / bC_l'$ is determined by the condition that the numerical value of the moment of the area under the curve of $cc_l\ A \cos^2\psi / bC_l'$ plotted against y must be equal to b^2.

The correction for the unyawed wing is

$$\Delta C_{D_i} = \frac{SC_L^2}{4by_1} \int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b \left(\frac{cc_l\ A}{bC_L}\right)_b dy \tag{10}$$

The correction to the induced drag coefficient for the yawed model may be written:

$$\Delta C_{D_i} = \frac{SC_L^2}{4b \cos\psi\ y_1} \int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b \left(\frac{cc_l\ A \cos\psi}{bC_L}\right)_b dy$$

$$+ \frac{S\ C_l' C_L}{2(|a_2|^2 - |a_1|^2)\cos^2\psi} \left[\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_l \left(\frac{cc_l\ A \cos\psi}{bC_L}\right)_b dy - \int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_t \left(\frac{cc_l\ A \cos\psi}{bC_L}\right)_b dy \right]$$

$$+ \frac{SC_l' C_L}{8y_1 b \cos^2\psi} \left[\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b \left(\frac{cc_l\ A \cos^2\psi}{bC_l'}\right)_l dy - \int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b \left(\frac{cc_l\ A \cos^2\psi}{bC_l'}\right)_t dy \right] \tag{11}$$

The parts of the drag correction caused by the interaction of the several components of dihedral upwash and dihedral load are neglected because these parts of the drag correction are small and tend to counteract each other.

Downwash.- The correction to the downwash angle at the tail location may be determined from the values of w/Γ for the tail location. (The boundary-induced wake displacement and the jet-effect of the pitched slipstream are neglected.)

$$\Delta\epsilon = 57.3\,\frac{V_o}{V}\left\{\left(\frac{w}{\Gamma}\right)_b\left(\frac{SC_L}{4y_1\cos\Psi}\right) + \frac{SbC_l{}'}{2\left(|a_2|^2 - |a_1|^2\right)\cos^2\Psi}\left[\left(\frac{w}{\Gamma}\right)_l - \left(\frac{w}{\Gamma}\right)_t\right]\right\} \qquad (12)$$

where all values of w/Γ are computed for the tail location. Equation (12) applies to either the yawed or the unyawed wing when the proper values of $\cos\Psi$ and $C_l{}'$ are substituted.

Pitching-moment coefficient.- The correction for the pitching-moment coefficient is determined from the corrections for the downwash and angle of attack and from the stabilizer effectiveness as

$$\Delta C_m = -\left(\frac{\partial C_m}{\partial i_t}\right)(\Delta\epsilon - \Delta\alpha) \qquad (13)$$

Rolling-moment coefficient.- The correction to the rolling-moment coefficient due to dihedral or to aileron deflection may be determined by similar methods if equal up-and-down aileron deflections were used. The general formula is

$$\Delta C_l{}' = \frac{-\,\partial c_l/\partial\alpha\; C_l{}'}{2\left(|a_2|^2 - |a_1|^2\right)\cos\Psi}\,\frac{A}{(A+4)}\int_{-b/2}^{b/2}\left[\left(\frac{w}{\Gamma}\right)_l + \left(\frac{w}{\Gamma}\right)_t\right] cy\,dy \qquad (14)$$

where $C_l{}'$ is the total dihedral or aileron wind-axis rolling moment and the factor $A/A+4$ approximately accounts for the effects of aerodynamic induction on the correction; that is, the rolling moment actually obtained on a twisted wing (twisted by the amount of the boundary-induced upwash angle) is approximately $A/A+4$ times the value calculated when two-dimensional flow conditions are assumed (reference 5). The factor $A/A+4$ applies strictly only if the wing is elliptical and the slope of the lift curve is equal to 2π. Because this factor is used as only a modification to a fairly small correction, it is seldom necessary to use more accurate values of the factor. If any other aileron-deflection ratio is used, the correction to the aileron rolling moment becomes

$$\Delta C_l{}' = \frac{-\partial c_l/\partial\alpha}{\left(|a_2|^2-|a_1|^2\right)\cos\psi}\,\frac{A}{A+4}\left[(C_l{}')_l\int_{-b/2}^{b/2}\left(\frac{w}{\Gamma}\right)_l cy\,dy+(C_l{}')_t\int_{-b/2}^{b/2}\left(\frac{w}{\Gamma}\right)_t cy\,dy\right] \tag{15}$$

where $(C_l{}')_l$ and $(C_l{}')_t$ are the components of aileron wind-axis rolling moment due to the ailerons on the leading and trailing wings, respectively.

<u>Yawing-moment coefficient</u>.- The correction to the induced-yawing-moment coefficient due to dihedral or to aileron deflection is determined by the same general methods that are used for the induced-drag correction. The interaction between the dihedral (or aileron) upwash and the dihedral (or aileron) loading will be neglected. The equation is

$$\begin{aligned}
\Delta C_{n_i} &= \frac{C_L{}^2}{4y_1A\cos\psi}\int_{-b/2}^{b/2}\left(\frac{w}{\Gamma}\right)_b\left(\frac{cc_lA\cos\psi}{bC_L}\right)_b y\,dy \\
&\quad-\frac{SC_L}{b\left(|a_2|^2-|a_1|^2\right)\cos^2\psi}\left[(C_l{}')_l\int_{-b/2}^{b/2}\left(\frac{w}{\Gamma}\right)_l\left(\frac{cc_lA\cos\psi}{bC_L}\right)_b y\,dy\right. \\
&\quad\left.+(C_l{}')_t\int_{-b/2}^{b/2}\left(\frac{w}{\Gamma}\right)_t\left(\frac{cc_lA\cos\psi}{bC_L}\right)_b y\,dy\right] \\
&\quad-\frac{C_L}{4y_1A\cos^2\psi}\left[(C_l{}')_l\int_{-b/2}^{b/2}\left(\frac{w}{\Gamma}\right)_b\left(\frac{cc_lA\cos^2\psi}{bC_l{}'}\right)_l y\,dy\right. \\
&\quad\left.+(C_l{}')_t\int_{-b/2}^{b/2}\left(\frac{w}{\Gamma}\right)_b\left(\frac{cc_lA\cos^2\psi}{bC_l{}'}\right)_t y\,dy\right]
\end{aligned} \tag{16}$$

All terms of the equation must be used for determining the correction for the yawing moment due to dihedral. The C_L^2 term resulting from the basic upwash velocity and basic loading need not be considered, however, in determining the correction to the increment of yawing moment caused by aileron deflection.

A correction to the yawing-moment coefficient due to the yawed slipstream and to the lateral force on the vertical tail may be determined from the jet-boundary-induced sidewash angle and the estimated slope of the normal-force curve for the vertical tail $\partial C_N/\partial\alpha$. The data of reference 6 may be used to estimate $\partial C_N/\partial\alpha$.

$$\Delta C_n = 57.3\left(\frac{v_s}{V} + \frac{v_f}{V}\right)\frac{\partial C_N}{\partial\alpha}\frac{S_f}{S_w}\frac{l}{b}\frac{q}{q_o} \tag{17}$$

NUMERICAL VALUES FOR A TYPICAL MODEL IN A 7- BY 10-FOOT CLOSED WIND TUNNEL

Model Dimensions

The dimensions of the model for which the jet-boundary corrections are calculated are given in the following table:

Wing area, S, square feet	10.36
Wing span, b, feet	7.8
Inboard tip aileron, $\lvert a_1\rvert$, feet	2.3
Outboard tip aileron, $\lvert a_2\rvert$, feet	3.74
Dihedral (from $\lvert y_1\rvert$ = 0 to 2.3 ft), degrees	0
Dihedral (from $\lvert y_1\rvert$ = 2.3 to 3.74 ft), degrees	6
Propeller diameter, feet	2
Aspect ratio, A	5.87
Taper ratio (rounded tips), λ.	0.66
Tail length (lifting line to three-quarter-chord tail), l, feet .	3.4
Ratio, area vertical tail to wing area, = S_f/S_w	0.10
Chord at plane of symmetry, c, feet	1.64
Chord 1.0 foot from plane of symmetry, c, feet	1.50
Chord 2.0 feet from plane of symmetry, c, feet	1.37
Chord 2.5 feet from plane of symmetry, c, feet	1.29
Chord 3.0 feet from plane of symmetry, c, feet	1.22
Chord 3.5 feet from plane of symmetry, c, feet	1.03
Chord 3.8 feet from plane of symmetry, c, feet	0.60
Chord 3.9 feet from plane of symmetry, c, feet	0
Assumed basic load horseshoe-type vortex semispan, $\lvert y_1\rvert$, feet . .	3.32

Wing and tail are assumed to lie on the horizontal plane of symmetry of the tunnel to simplify the calculations.

Jet-Boundary-Induced Upwash Velocity

The jet-boundary-induced upwash velocity at the tail and at the lifting line for horseshoe-type vortices of unit strength representing the basic loading, the aileron or dihedral load on the leading wing, the aileron or dihedral load on the trailing wing, and also for a dihedral load completely across the leading wing was calculated from equations (1) to (3) for the model yawed 20° in a 7- by 10-foot closed wind tunnel. The values of the integers n and m used in the summation are those found to be important from the calculations of reference 3. The upwash-velocity values for the aileron loading are also used as upwash-velocity values for the loading due to the partial-span dihedral which extends across the aileron span. The values of w/Γ calculated for dihedral extending completely across the leading wing are presented but are not used in any of the following calculations.

The upwash velocities for the several loadings for the model yawed 20° are presented in figure 2. The abscissa y is for the body-axis system. For comparison the upwash velocities for the same loadings for the unyawed model are included in figure 2. Owing to the great amount of calculations involved, the upwash velocity for the unyawed model was not, however, computed from the same equations and in the same manner as was the upwash velocity for the yawed model. The usual simple summation formulas for the lifting-line upwash velocity of the unyawed model were used (reference 2) and the values of the upwash behind the lifting line were obtained from reference 3. The comparison, therefore, is not strictly valid but a check indicated that the two methods of calculation are in fairly good agreement.

Jet-Boundary-Induced Sidewash Angle

Slipstream.- The value of the summation factor of equation (4) for a 7- by 10-foot closed tunnel is 0.66 for unit slipstream radius. Equation (4) may be simplified and rewritten as

$$\frac{v_s}{V} = 0.66 \frac{\frac{q_s}{q_o} - 1}{\frac{q_s}{q_o} + 1} \frac{V_o}{V} \sigma_s$$

for unit slipstream radius such as is the case for this model. The value of q_s/q_o for operation of the propeller of this model for rated-power conditions may be approximated by the linear equation $q_s/q_o = 1 + 0.6\ C_L$. The value of V_o/V is $\sqrt{\dfrac{1}{1 + 0.6C_L}}$ if the tail is in the slipstream or unity if the tail is outside the slipstream.

The sidewash correction and also the downwash and pitching-moment corrections at $\psi = 20^\circ$ will be computed for two extreme cases. In one case the vertical tail is assumed to be completely immersed in the slipstream; in the other, it is assumed to be completely free of the slipstream. The normal condition will usually be somewhere between these two extremes; for the tail generally begins to move out of the slipstream at about 15° angle of yaw and is completely out of the slipstream at $\psi = 25^\circ$ for a conventional single-engine airplane. The values of V_o/V for the two conditions and the corresponding values of the jet-boundary-induced sidewash angle v_s/V in radians are shown in figure 3.

Vertical tail.- The value of δ from reference 2 is about 0.14; therefore,

$$\frac{v_f}{V} = \delta\ \frac{S_w}{C}\ C_Y{}'\ \frac{V_o}{V} = 0.14\ \frac{10.36}{70}\ C_Y{}'\ \frac{V_o}{V}$$

If $C_Y{}'$, as determined experimentally, is 0.14 at $\psi = 20^\circ$, values of v_f/V as presented in figure 4 are obtained for the two slipstream conditions.

Corrections

Angle of attack.- A mechanical integration of the w/Γ curves of figure 2 gives for the wing at 0° yaw

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b dy = 0.1770$$

and for the wing at 20° yaw, if the dihedral is over only the aileron sections,

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b dy = 0.1768$$

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_l dy = 0.0478$$

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_t dy = 0.0373$$

Substitution of the numerical values in equations (6) and (7) gives for the unyawed wing

$$\Delta\alpha = \frac{10.36 \times 57.3}{4 \times 3.32 \times 7.8} C_L \ (0.1770) = 1.02 \ C_L$$

and for the yawed wing ($\cos 20° = 0.94$)

$$\Delta\alpha = \frac{10.36 \ C_L \times 57.3 \ (0.1768)}{4 \times 3.32 \times 7.8 \times 0.94} + \frac{10.36 \times 57.3 \ C_l' \ [(0.0478)-(0.0373)]}{(2.94)^2 \ [(3.74)^2-(2.3)^2]}$$

$$\Delta\alpha = 1.08 \ C_L + 0.41 \ C_l'$$

For $\psi = 20°$, $C_l' = 0.012$ for this model with 6° dihedral over aileron sections; therefore,

$$\Delta\alpha = 1.08 \ C_L + 0.0049$$

<u>Induced-drag coefficient</u>.- The span-loading curves for the basic loading and for the aileron loading (to be used also as the dihedral loading) as obtained from reference 4 and converted to the parameters $cc_l A \cos \Psi / bC_L$ and $cc_l A \cos^2\Psi / bC_l{}'$ are given in figure 5. The product of the upwash velocity and the loading parameter was calculated and mechanically integrated to give, for $\Psi = 0°$,

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b \left(\frac{cc_l A}{bC_L}\right)_b dy = 0.1717$$

and for $\Psi = 20°$

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b \left(\frac{cc_l A \cos \Psi}{bC_L}\right)_b dy = 0.1717$$

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_l \left(\frac{cc_l A \cos \Psi}{bC_L}\right)_b dy = 0.0460$$

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_t \left(\frac{cc_l A \cos \Psi}{bC_L}\right)_b dy = 0.0346$$

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b \left(\frac{cc_l A \cos^2 \Psi}{bC_l{}'}\right)_l dy = 0.5368$$

$$\int_{-b/2}^{b/2} \left(\frac{w}{\Gamma}\right)_b \left(\frac{cc_l A \cos^2 \Psi}{bC_l{}'}\right)_t dy = 0.6960$$

Substitution of the proper numerical values in equations (10) and (11) gives, for the unyawed wing,

$$\Delta C_{D_i} = 0.0172 C_L{}^2$$

and, for the wing yawed 20°,

$$\Delta C_{D_i} = 0.0182 C_L{}^2 - 0.0013\ C_l{}'\ C_L$$

and if $C_l{}'$ due to dihedral for the wing yawed 20° equals 0.012,

$$\Delta C_{D_i} = 0.0182 C_L{}^2 - 0.00002\ C_L$$

Downwash.- Values of w/Γ at the tail location are given in figure 2. Substitution in equation (12) gives, for the model at $\psi = 0°$,

$$\Delta\epsilon = 1.67\ \frac{V_o}{V}\ C_L$$

and, for the model at $\psi = 20°$,

$$\Delta\epsilon = (1.74\ C_L + 2.27\ C_l{}')\ V_o/V$$

With a value for $C_l{}'$ of 0.012 for the model at $\psi = 20°$

$$\Delta\epsilon = (1.74\ C_L + 0.027)\ V_o/V$$

Pitching-moment corrections.- The corrections $\Delta\alpha$ and $\Delta\epsilon$ and the stabilizer effectiveness $\partial C_m/\partial i_t$ are used in determining the pitching-moment corrections. The pitching-moment corrections given in figure 6 are obtained by substituting in equation (13) the values of the previously determined correction $\Delta\alpha$ and $\Delta\epsilon$, values of $\partial C_m/\partial i_t = -0.023$, power off, and $\partial C_m/\partial i_t = -0.023-0.0138\ C_L$, power on, and the previously determined V_o/V ratio.

Rolling-moment corrections.- From equation (15), if a value for $\partial c_l/\partial \alpha$ of 6.0 per radian is used and if the moments are determined by mechanical integration, the aileron rolling-moment correction at $\psi = 0°$ is

$$\Delta C_l{}' = -0.052 \frac{6.0\ C_l{}'}{(3.74)^2-(2.3)^2} = -0.036\ C_l{}'$$

and at $\psi = 20^\circ$

$$\Delta C_l{}' = -0.030\ (C_l{}')_l \pm 0.032\ (C_l{}')_t$$

and the dihedral rolling-moment correction is

$$\Delta C_l{}' = -0.031\ C_l{}'$$

since

$$(C_l{}')_l = (C_l{}')_t = \frac{C_l{}'}{2}$$

<u>Yawing-moment corrections</u>.- Substituting the numerical values in equation (16) and integrating gives the correction to the yawing-moment coefficient due to aileron deflection for $\psi = 0^\circ$

$$\Delta C_{n_i} = -0.0279\ C_l{}'\ C_L$$

and for $\psi = 20^\circ$

$$\Delta C_{n_i} = -\left[0.0256\ (C_l{}')_l + 0.0332\ (C_l{}')_t\right] C_L$$

The correction to the yawing-moment coefficient due to yaw (dihedral and basic load) at $\psi = 20^\circ$ ($C_l{}' = 0.012$) is

$$\Delta C_{n_i} = 0.00044\ C_L{}^2 - 0.00035\ C_L$$

From equation (17) and from values of v_s/V in figure 3, v_f/V in figure 4, $S_f/S_w = 0.10$, $l/b = 0.44$, $\partial C_N/\partial\alpha \approx 0.045$ (from reference 6)

$$\Delta C_n = 0.045 \times 0.10 \times 0.44 \times 57.3 \left(\frac{v_s}{V} + \frac{v_f}{V}\right)\left(\frac{q}{q_o}\right)$$

$$= 0.114 \left(\frac{v_s}{V} + \frac{v_f}{V}\right)$$

Approximating the v_s/V and v_f/V curves of figures 3 and 4 by linear equations and adding the ΔC_n correction due to basic load, dihedral load, slipstream, and vertical-tail load give a total yawing-moment-coefficient correction at 20° yaw (excluding the effect of the ailerons) of approximately

$$\Delta C_n = (0.0004\ C_L^2 - 0.0004\ C_L) + (0.0005\ C_L + 0.0003)$$

$$= 0.0004\ C_L^2 + 0.0001\ C_L + 0.0003$$

CONCLUDING REMARKS

The calculations indicate that, for this model-tunnel combination and for a constant immersion of the tail in the slipstream, the corrections to the angle of attack, induced-drag coefficient, pitching-moment coefficient, and downwash are about 6 percent greater at 20° yaw than at the same lift coefficient at 0° yaw. The lift coefficient is usually between 6 and 12 percent lower at 20° yaw than at 0° yaw, because the lift usually decreases at a rate somewhere between the cosine and the cosine squared of the angle of yaw (reference 7). Thus, for practical purposes, the same numerical values of the correction increments may generally be used for the yawed model at the same angle of attack as for the unyawed model, because the lift coefficient decreases with yaw approximately as the correction factor increases with yaw.

The effect of the slipstream location with respect to the tail upon the downwash and pitching-moment correction is, however, much more important. The value of the stabilizer effectiveness $\partial C_m/\partial i_t$ may be used as a measure of the immersion of the tail (reference 3) in estimating the ratio of the local velocity at the tail to the free-stream velocity. If values of the boundary-induced upwash velocity at the lifting line and behind the lifting line of the unyawed model are used together with the values of the local velocity ratio at the tail of the yawed model, the computed downwash and pitching-moment coefficient corrections are in fairly good agreement with the values computed from boundary induced upwash velocities for the yawed model.

The rolling-moment coefficient correction for the model at 20° yaw is approximately 15 percent lower than the correction for the unyawed model. The correction to the aileron yawing-moment coefficient for the aileron on the leading wing of the yawed model is 8 percent lower than that for the unyawed model, but the correction for the aileron on the trailing wing is about 20 percent greater than the correction for the unyawed model. The correction at unit lift coefficient and at 20° yaw to the yawing-moment coefficient of the complete model with or without power is about 5 percent of the yawing-moment coefficient of the model.

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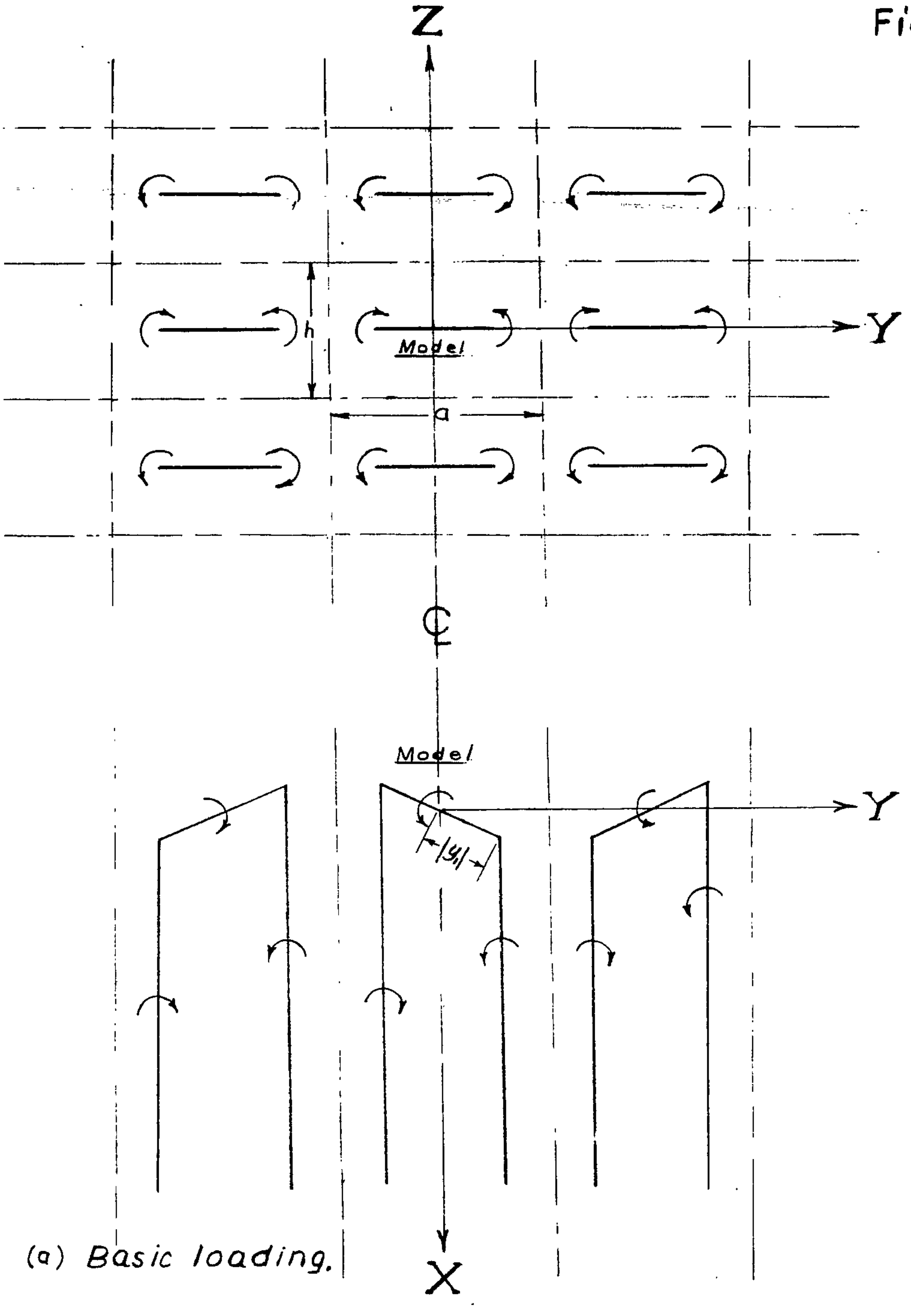
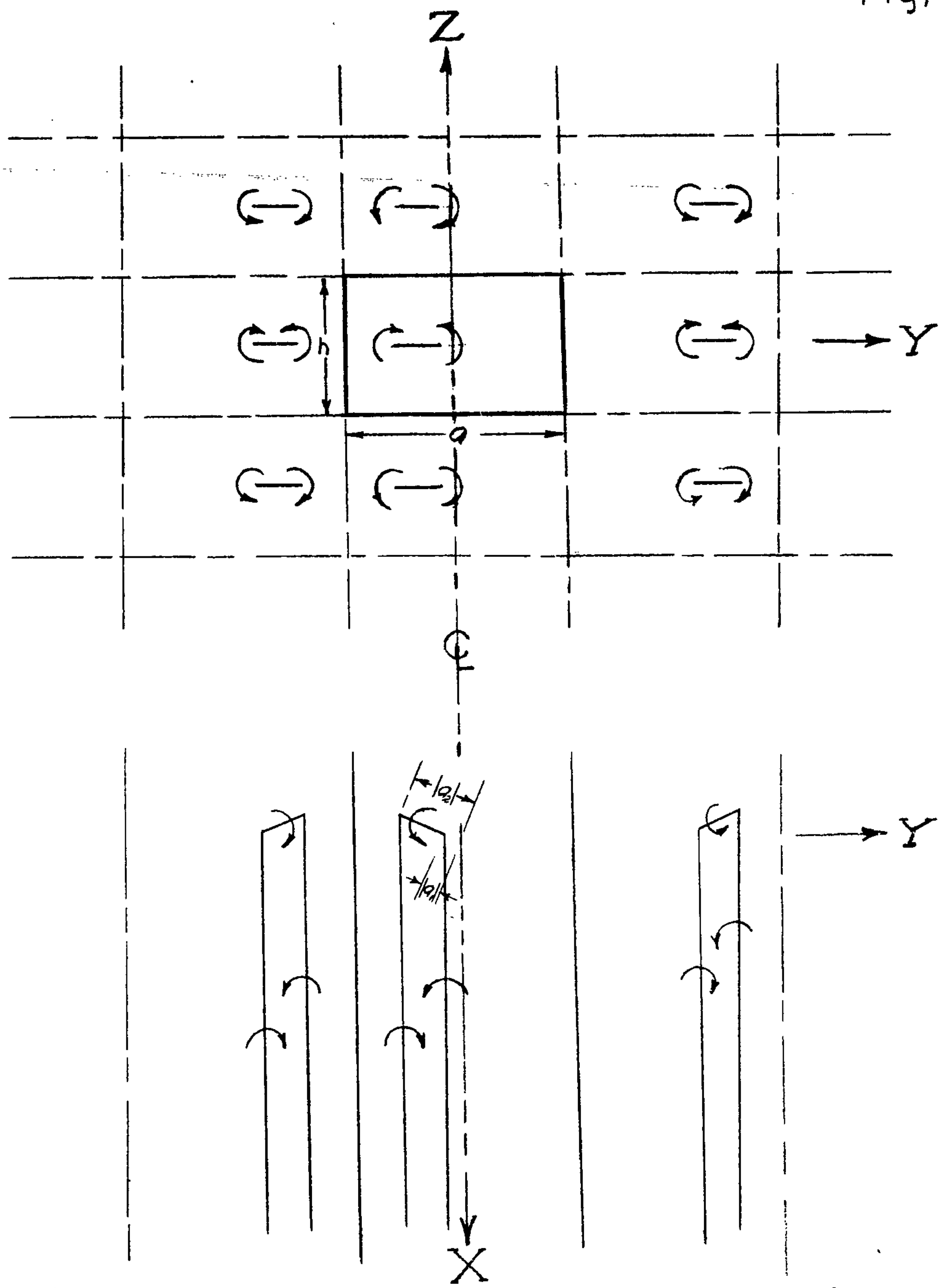


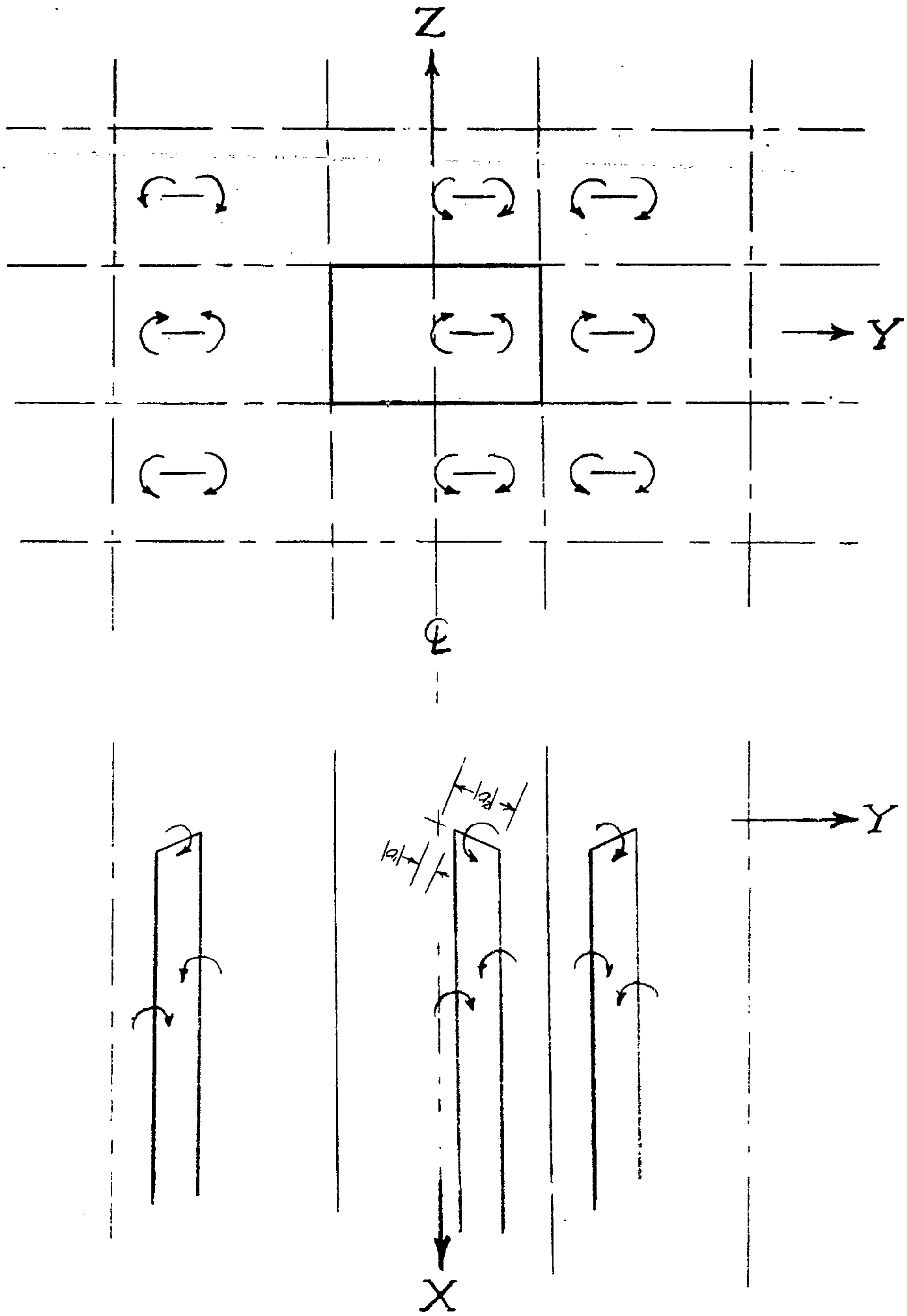
(a) Basic loading.

Figure 1.- Doubly infinite image system to satisfy the boundary conditions for a yawed model in a closed rectangular wind tunnel.



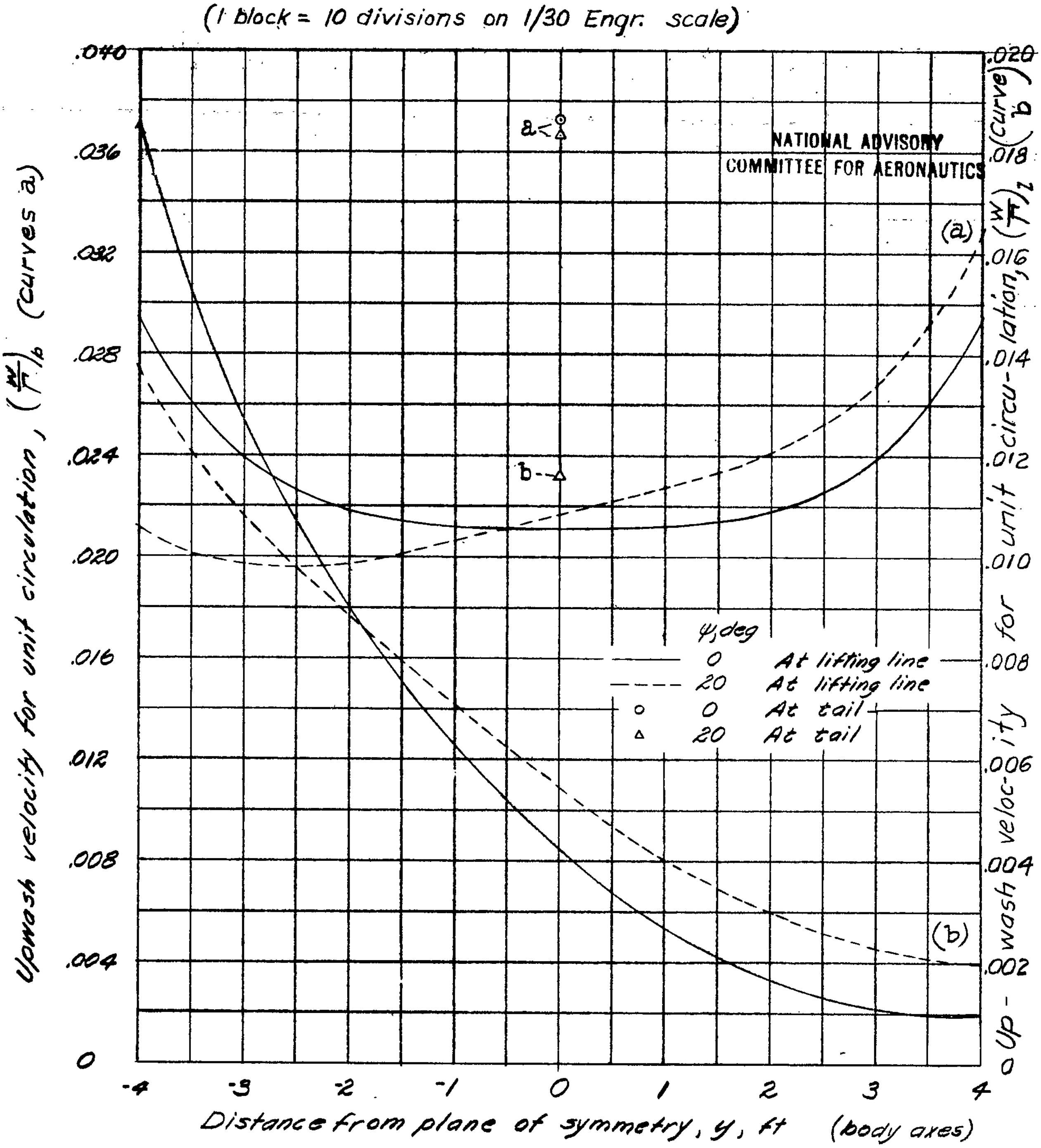
(b) *Aileron or dihedral loading on leading wing.*

Figure 1. - Continued.



(c) Aileron or dihedral loading on trailing wing.

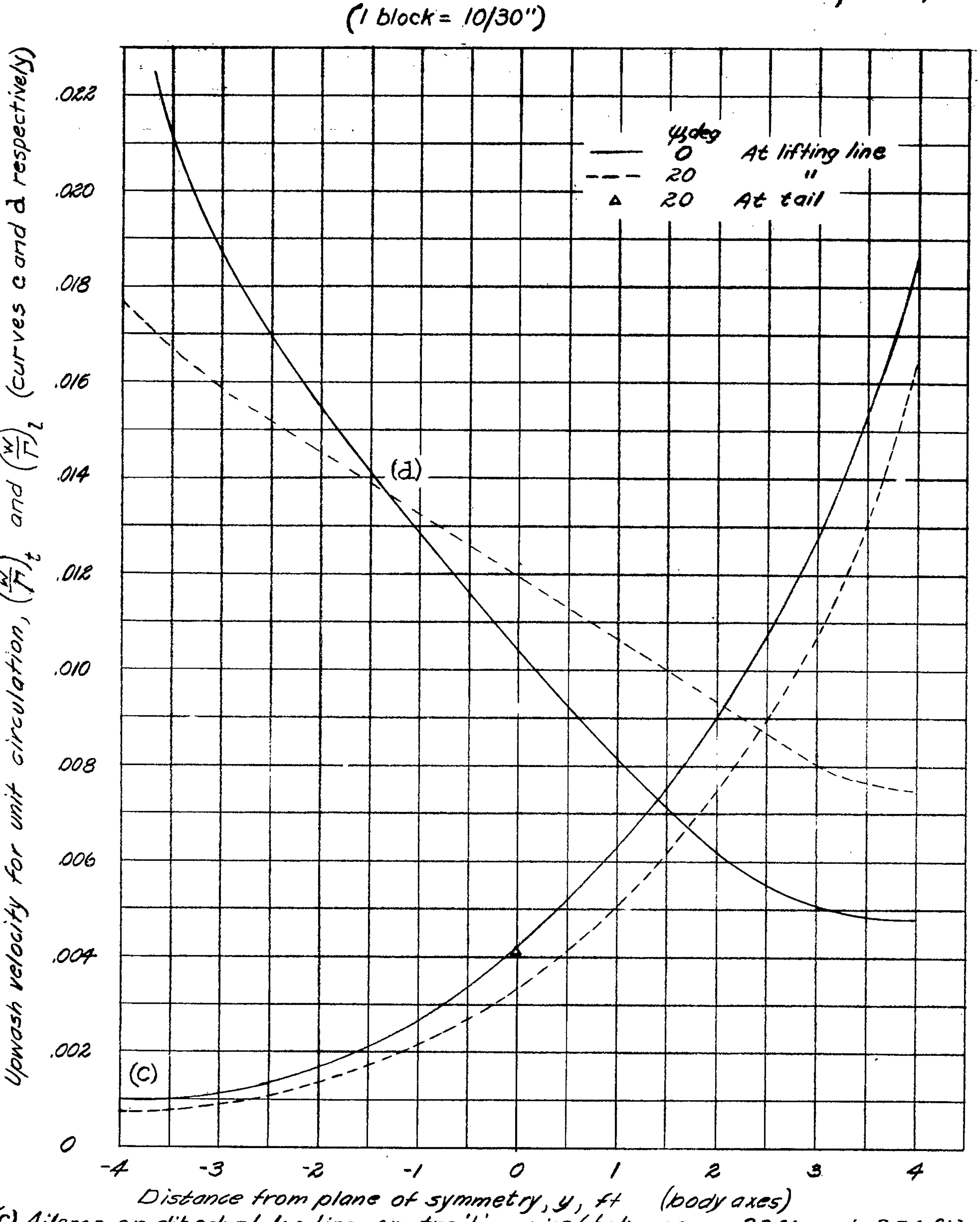
Figure 1. — Concluded.



(a) Basic loading ($|y_1|$ = 3.32 ft).

(b) Aileron or dihedral loading on leading wing (between y_1 = -2.3 ft and -3.74 ft).

FIGURE 2.- Jet-boundary induced-upwash velocity for a model in a 7- by 10-foot closed wind tunnel.



(c) Aileron or dihedral loading on trailing wing (between y_1 = 2.3 ft and 3.74 ft).
(d) Aileron or dihedral loading on leading wing (between y_1 = 0 and -3.32 ft).

FIGURE 2.- Concluded.

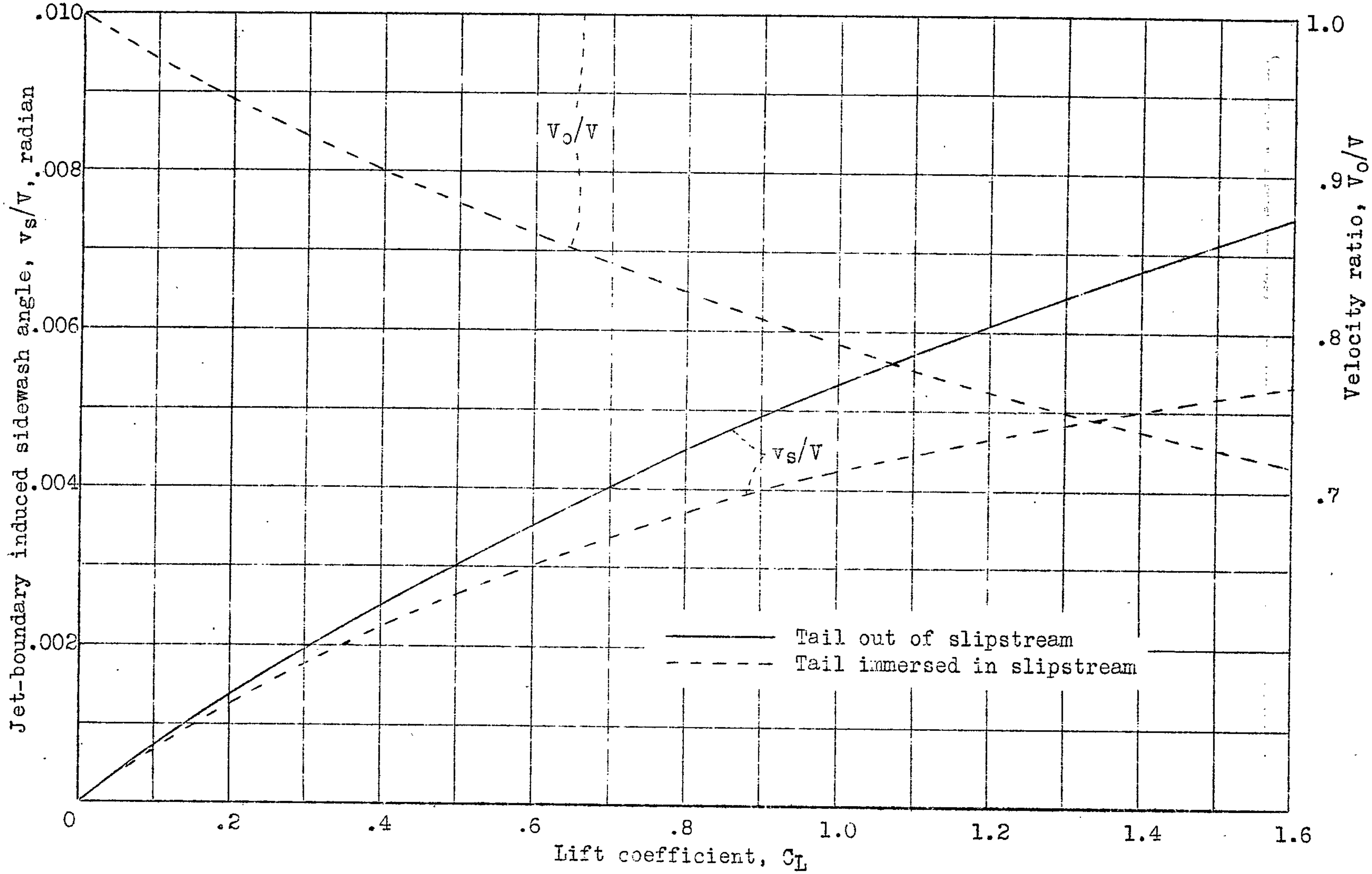


Figure 3.- Values of the velocity ratio, V_o/V, and the jet-boundary induced sidewash angle, v_s/V, for a model in a 7-by 10-foot closed wind tunnel.

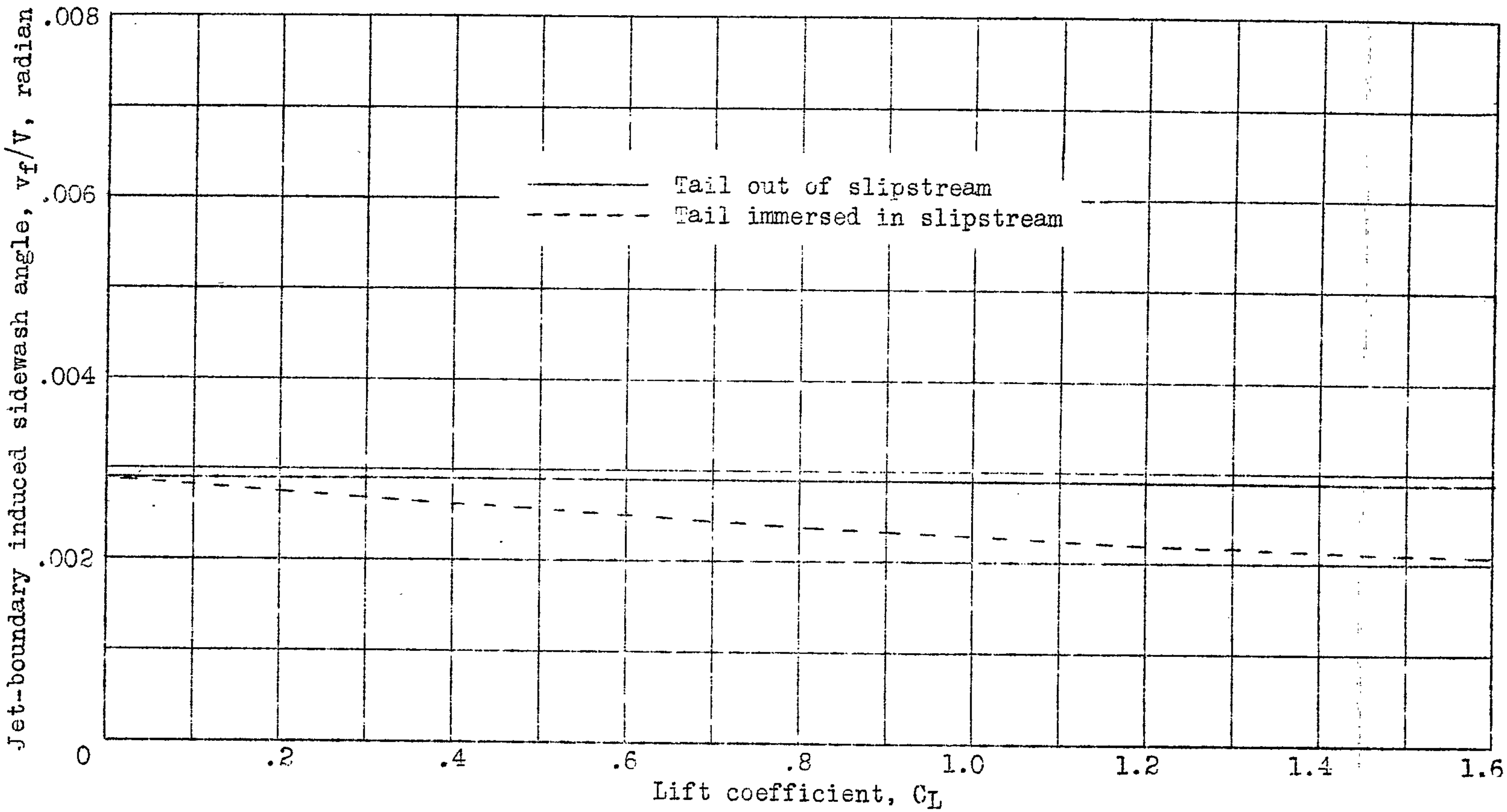
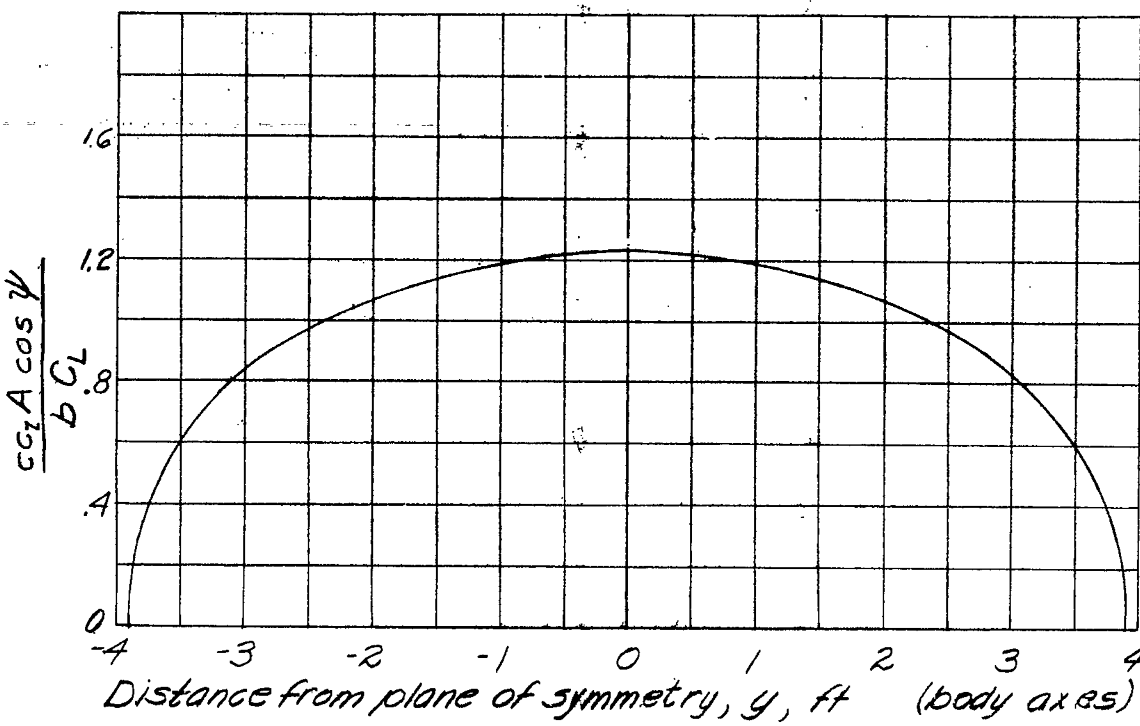
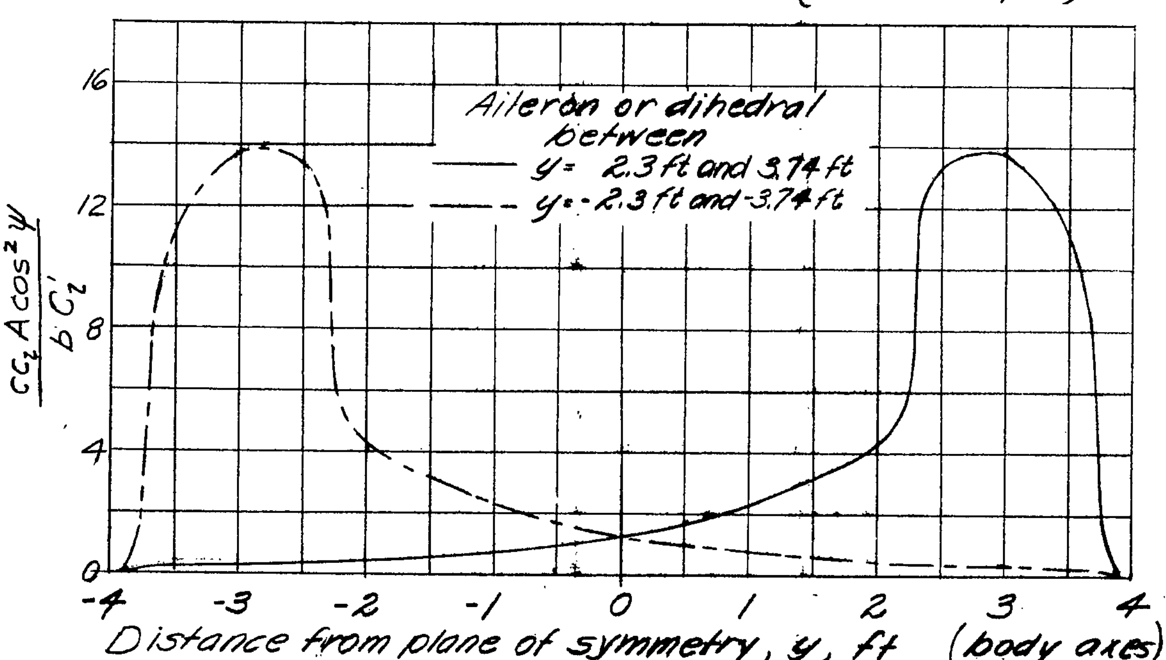


Figure 4.- Values of the jet-boundary induced sidewash angle, v_f/V, for a model in a 7-by 10-foot closed wind tunnel.



(a) Basic loading.

(1 block = 10/30")



(b) Aileron or dihedral loadings.

FIGURE 5.- Span load distributions; A, 5.87; λ, 0.66.

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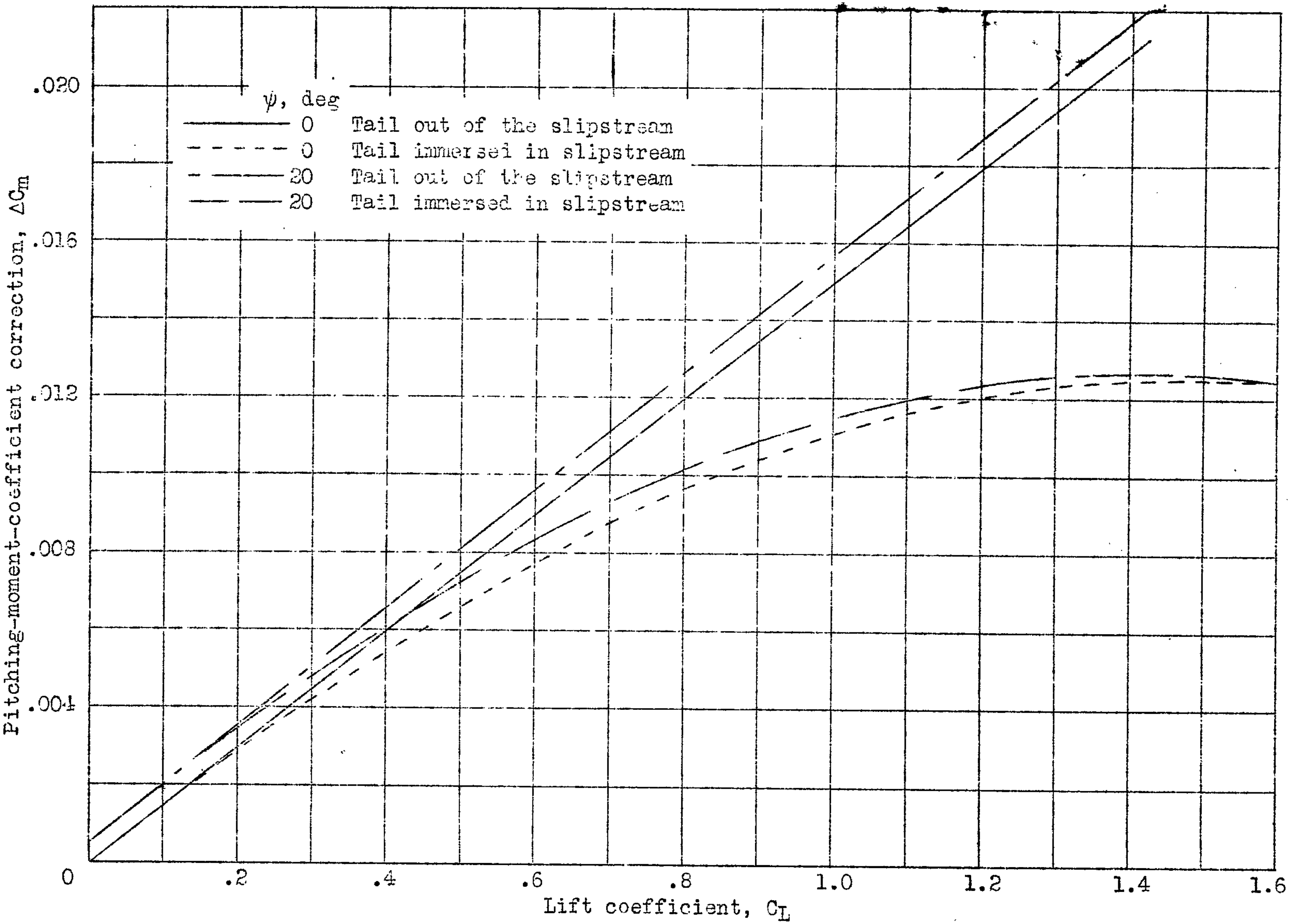

Fig. 6

Figure 6.- Jet-boundary correction to the pitching-moment coefficient for a model in a 7-by 10-foot closed wind tunnel.